Auctions with Reference Prices: An Empirical Analysis of Tea Auctions in Bangladesh

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Abstract

Auctions for sale of agricultural produce have been gaining popularity in recent years. This paper uses a game-theoretic model of behaviour within the independent private-values paradigm (IPVP) to put structure upon a unique dataset collected from one of the seven brokerage firms that conducts auctions for sale of tea in Bangladesh. Incorporating insights from the theory of auctions, this paper estimates the distribution of valuation for the bidders in the Chattogram tea auction, both parametrically and non-parametrically. This flexible, realistic empirical framework enables the study of the equilibrium relationship between bidding behaviour and sale prices in the tea auctions. The results from counterfactual exercises suggest that the existing mechanism of oral, ascending price auction, actually would garner higher revenue for the auctioneer compared to a Dutch auction. On the other hand, implementation of a Generalized Vickrey Auction (GVA) in case of multi-unit sale would generate substantially higher revenue than single-unit English auctions. This paper also estimates the optimal reserve price for the auctioneer as well as the average surplus for the bidders.

JEL: C57, D44, L11, Q13.

Keywords: tea market, English auction, optimal reserve price, structural estimation of auctions.

1 Introduction

Auctions have existed since ancient times. It is one of the oldest market mechanisms for exchange of goods and services through competitive bidding. Today, a variety of agricultural

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goods and natural resources, including tobacco, tea, lumber, fish, and oil are sold through auctions. [1] Even though the rules may vary across different types, auctions perform a critical service referred to as price discovery. Hence, economists have long been studying auctions as games of incomplete information originating from Vickrey (1961).

The sale of manufactured tea in Bangladesh to wholesalers is an important application of the auction mechanism for allocation of the produce. The bulk of the produced tea in Bangladesh, is sold in the open market via oral, ascending price auctions. Each year roughly 80 million kilograms of tea are sold through Chattogram tea auctions generating a revenue of approximately $153 million. This nearly 75-year-old auction market offers a rich setting for analysing auctions.

In this study, I use a game-theoretic model of behaviour within the independent private-values paradigm (IPVP) to put structure upon a unique dataset of tea auctions held in Chattogram, Bangladesh. I collect data from one of the seven brokerage firms that conducts the tea auctions. On an auction day, this auctioneer, like others, sells many lots of tea via sequential English auctions. The dataset includes detailed information on the tea lots: the winning bid for each lot, the number of bidders, the appraised value posted by the auctioneer etc.

Within the IPVP framework, the equilibrium bidding strategies of potential bidders are increasing functions of their valuations. At the English auctions (applied in the Chattogram tea auctions), the dominant bidding strategy for bidders who lose the auction is to bid their valuations. Using this principle, one can estimate the underlying probability law of valuations using the empirical distribution of winning bids from a cross-section of auctions (see Athey and Haile (2002)). I estimate the distribution of valuation for the bidders in the Chattogram tea auctions both parametrically and non-parametrically.

A principal advantage to estimating a structural model using auction theory is that it enables simulation of alternative states of the world. I use my estimation results to conduct several important counterfactual exercises. For example, I use my estimates to compare the revenues a seller could expect to earn were a Dutch auction employed instead. My results suggest that English auctions would generate slightly higher revenue compared to Dutch auctions. Thus, the much celebrated "Revenue Equivalence Proposition", which suggests that on average a seller would get same revenue no matter whichever of these two formats are

[1] Hubbard and Paarsch (2015) details many applications of auctions

used (see Milgrom (2004) for details), does not seem to hold in Chattogram tea auctions.

Riley and Samuelson (1981) have shown that within the IPVP framework, designing a selling mechanism which maximizes the seller's expected revenue involves choosing the reserve price, the minimum price that must be bid, optimally. The optimal selling mechanism depends upon the distributions of valuations of the bidders. Although there is no reserve price set by the auctioneers in the the Chattogram tea auctions, I have used the estimated distribution of valuations from my model to calculate the optimal reserve price.

As mentioned earlier, the Chattogram tea auctions are sequential English auctions where multiple units (or lots) are auctioned off. Hence, I also extend my empirical results derived from the case of single-unit actions to the case of multiple units. I then test if the generalized Vickrey auction (GVA) which is the multi-unit analogue of the second-price (Vickrey) auction would generate more revenue for the auctioneer. In this case, it is found that the average winning bid in case of the simulated generalized Vickrey auctions would garner substantially higher revenue for the auctioneer.

Contribution to the literature: This paper contributes to several distinct literatures. The first comprises analyses of the Chattogram tea auctions: an important part of the tea industry of Bangladesh, whose market value in 2021 was $350 million. The use of auctions to effect sales of tea in wholesale markets has a long history in Bangladesh. However, studies analyzing the process of price formation in this market do not exist. Incorporating insights from the theory of auctions, this paper builds a flexible, realistic empirical framework which enables the study of the equilibrium relationship between bidding behaviour and sale prices in Chattogram tea auctions.

My paper also contributes to the vast literature on empirical auction estimation (see Athey and Haile (2002) for a survey). Following Paarsch (1997), I use the information on the winning bids from auctions to back out a parametric distribution of the valuation of the bidders. Also, I estimate the distribution non-parametrically. Furthermore, my counterfactual exercises include simulating auctions in a sequential multi-unit setting.

This paper also contributes to the literature on agricultural marketing. Research on agricultural markets that uses the structural auction framework is relatively limited. Athey and Levin (2001) studied the U.S. Forest Service timber auctions. Banerji and Meenakshi (2004) employed auction theory to analyze wholesale markets for wheat in Northern India. However, auctions for agricultural produce have been a popular market-research topic among agricultural economists (Sogn-Grundvåg and Zhang (2021), Levi et al. (2020), Prdić and Kuzman (2019)). One could extend the model proposed in this paper to study auctions used in other agricultural produce and market settings.

The rest of the paper proceeds as follows. Section 2 provides an overview of the Bangladesh tea industry. Section 3 describes the mechanism of the Chattogram tea auction. I describe my dataset and present summary statistics in section 4. Reduced form regression results are provided in section 5. Section 6 describes the model and shows the results from parametric estimation. Section 7 presents the results from non-parametric estimation. I present the results from different counterfactual exercises in sections 9-12. I conclude with a brief discussion in Section 13.

2 Tea Industry of Bangladesh

This section provides a broad overview of the tea industry of Bangladesh. The data and statistics mentioned in this section are collected from the website of the Bangladesh Tea Board [2], a statutory body constituted under the Tea Ordinance 1977 of Bangladesh to regulate, control and promote the cultivation and sale of tea in Bangladesh.

Tea is considered the most popular and affordable drink in Bangladesh. It is the second largest cash crop in Bangladesh after jute. Bangladesh is amongst the leading tea producer in the world. Currently, Bangladesh accounts for roughly 3% of global tea production. In 2021, the market size of the tea industry in Bangladesh was about $350 million.

Tea cultivation in Bangladesh began during the British rule. The first commercial-scale tea garden in Bangladesh, 'Malnichhara Tea Garden,' was established in Sylhet in 1854. Tea used to be cultivated only in two districts: the "Surma Valley" in Sylhet and the "Halda Valley" in Chattogram. Since 2002, tea has also been cultivated in Panchagarh, Lalmonirhat, Thakurgaon, Nilphamari, Dinajpur, and Bandarban districts. The country has 167 tea gardens that cover 113112 hectors (about 280,000 acres) of granted land. A total of 359,085 people live in tea garden areas, including 89,812 registered workers and 19,592 casual workers working in the tea gardens. Most of the workers are not of local origins but are mainly descendants of immigrants who came from different parts of India during British rule. Figure 1 shows the

[2] please see http://www.teaboard.gov.bd/

tea-producing districts of Bangladesh.

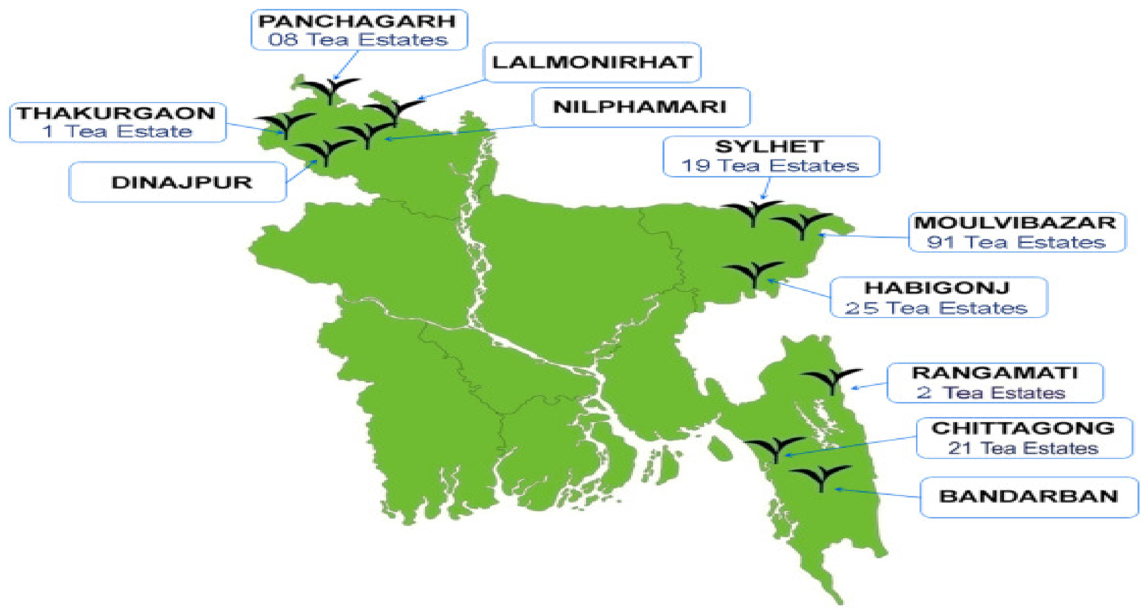


Figure 1: **Tea-producing Districts of Bangladesh**

The Bangladesh tea industry has seen steady growth in production in the last three decades. Tea production in 1980 stood at 31.37 million kgs which rose to 96 million kgs in 2021, which is more than a threefold increase in four decades. It is currently the ninth largest producer of tea in the world. Bangladesh globally exported 680,000 kgs of tea and earned $180.57 million in 2021. The domestic consumption of tea is increasing at an average rate of 4.61% per annum, resulting in decreased export earnings over the years. Table 1 gives the important statistics of the industry in 2021.

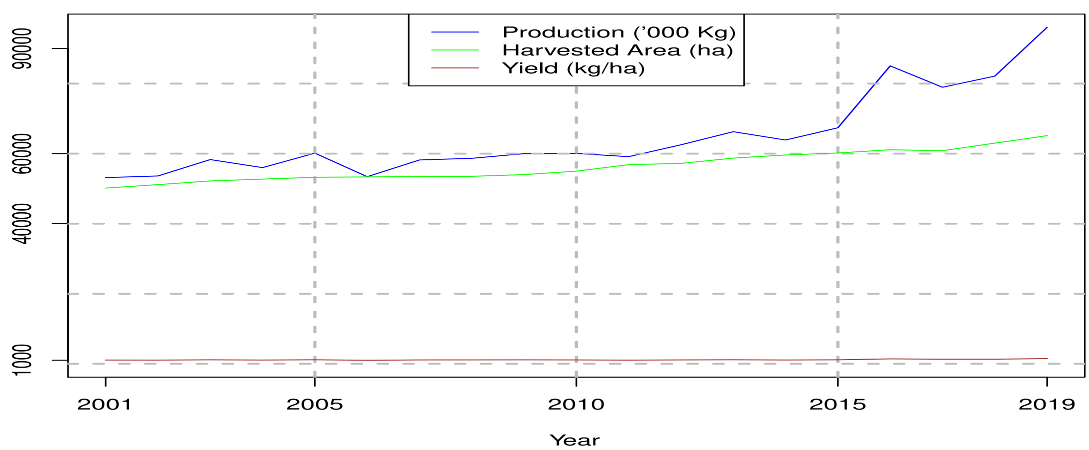
Table 1: **Bangladesh Tea Industry in 2021** [a]

Number of Tea Gardens	167	Export	0.68 million ton
Grant Area	113112.42 ha	Domestic Consumption	95.20 million ton
Harvest Area	62993 ha	Total Workers	109404
Production	96.5 million ton	Yield Rate	1475.12 kg/ha
Growth Rate	1.89%	Avg. Retail Price	477 Tk/kg

[a] Data collected from Bangladesh Tea Board.

The harvest area, production, and yield of tea production over the last 20 years are displayed in Figure 2. Production has increased steadily over the years at a compound annual growth rate of 2.65%. The growth of the production has mostly been driven by the increase in the harvest area, which grew at a rate of 1.31%. The yield, measured as production per hector of land, has increased at a rate of 1.33%. Saha et al. (2021) suggest that when production technology was mostly traditional, most of the increase in output came from a larger harvest area. When technology started to get better, most of the increase in output resulted from a higher yield.

Figure 2: **Growth Trend of Bangladesh Tea Production**



In Bangladesh, the marketing system for tea is well institutionalized and is different from that of other agricultural products. The parties involved in various stages of tea marketing require specific qualifications. The tea estates, brokers, bidders/blenders, wholesalers, and retailers are the participants involved in the tea marketing system. Tea sales in Bangladesh are generated mostly through auctions, which are held weekly in Chattogram and Sreemongol. Tea auctions were established in 1949 by British and Australian traders, and the tradition has continued to this day.

3 Tea Auction in Chattogram

In the domestic market, tea is sold in two ways: direct sale from the estates and sale by auction. The tea estates can sell a limited quantity of tea directly in the local market, as approved by the Bangladesh Tea Board. An estate can sell upto 20% of its total production by this method. Before selling directly to the market, the estate must take permission from Bangladesh Tea Board (BTB). They have to mention the quantity and grade of tea to be sold. After getting the permission, the estate sells to the buyers after adding the Value Added Tax (VAT) to the agreed price.

In Bangladesh, the bulk of the produced tea is sold in the open market via auctions. Currently, there are two auction centers: one in Chattogram and the other in Sreemangal. The Chattogram auction house accounts for more than 97% of the auction sale. The auctions are organized by the Tea Association of Bangladesh (the association of tea estate owners) and are administered by seven auction houses or brokers in the same venue in the city of Chattogram. There are usually 45 auctions every year, with an auction held on each Monday during the months of April to February except for the two Muslim religious holidays. Table 2 shows the annual sales and average price (per kg) in Chattogram tea auction center over the last six years.

Table 2: **Annual Sales and Average Price in Chattogram Tea Auction Center** [a]

Season (April-March)	Quantity (Million Kg)	Average Price/Kg (BDT)
2015-2016	62.67	187.09
2016-2017	72.28	191.01
2017-2018	76.57	214.10
2018-2019	79.34	262.96
2019-2020	90.44	176.08
2020-2021	82.58	189.07

[a] Data collected from Bangladesh Tea Board.

Tea sold through auction follows the following timeline. First, tea from various tea estates of the country are sent to the 16 warehouses in Chattogram city. A tea estate prepares an invoice for the designated warehouse which includes information like number of tea lots (chests), quality of tea, grade of tea etc. The tea to be sold on a specific auction day is needed to entered in the catalog of a particular auctioneer/broker almost two weeks in advance. Once received by the warehouse, the auctioneer, chosen by the tea estate, takes control.

The warehouse sends one copy of the invoice form to the selected broker. The broker then goes to the warehouse and sets aside 1.8 kg of tea from each lot of tea to distribute as samples among the auctioneer as well as the buyers. A lot typically contains 10 or 20 bags of identical weight (usually around 50–55 kg). After collecting samples, the brokerage company tastes the tea by palate in order to assess the expected sale price. The price of the tea is determined on the basis of quality, colour, appearance, market demand, etc.

The auction catalog prepared by a specific auctioneer for a specific auction day lists the sequence of the lots of tea that he will put up for sale on that day and typically includes all the tea that he has in the warehouses, received from various tea estates. The catalog contains the name of the estate, lot number, quantity, appraised value (or as mentioned by the brokers, ″valuation″), warehouse address, etc. for each lot of tea. The final catalog is sent to registered bidders (currently 37 in total) along with tea samples from each lot, typically five days before each auction. Each interested bidder then independently determines the quality of the tea after tasting by his own tea-taster. These bidders are generally wholesalers.

On the auction day, each of the seven auctioneers sells their lots in turn. The sequence of auctioneers in the first auction day of the year is decided by lottery. This sequence is changed every week, with the first auctioneer from the last auction moving to the seventh spot and all the other auctioneers moving up one spot. During his turn, an auctioneer uses English auctions to sell his lots in the order they are listed in the catalog.

At the auction, the price per Kg of tea in the lot is determined. The bid opens at the ″valuation″ price mentioned in the catalog. However, any bidder is free to bid a price lower than the appraised value listed in the catalog. Also, there is no specific rule on ′bid increment′. Bidders are free to choose the increment when they bid. All lots are subject to a hidden reserve price, unless expressly stated to the contrary or declared by the auctioneer when the lot is first put up for the sale. If such price is not offered, the lot is withdrawn and is placed on a list of ′out lots′.

The amount of time an auctioneer takes to sell a lot is 15-60 seconds. After he sells all the lots on his catalog, the next auctioneer sells the lots on his catalog. The day of the auction is over when all seven auctioneers have sold all the lots on their catalogs. The buyer pays the price and takes delivery of the lots purchased by him on or before the prompt day, which will be 13 days after the sale (auction day), unless otherwise advised by the Tea Traders Association of Bangladesh. As a commission, the auctioneer/broker gets 1% of the sale price from the tea estate and Tk. 0.05/Kg from the buyer of each lot, no matter how much the lot was sold for.

4 Data

In this section, I describe the data used for the study. I collected catalog data from one of the largest auction brokers for 26 auction days from November 2021 to July 2022. In total, I have detailed data on the sale of 2495 lots. This auction house accounts for more than 7% of the total tea sold through auction in Chattogram. The auction house is a pure auctioneer. It only sells tea lots from estates owned by client sellers. Some other auctioneers, who are referred to as 'integrated auctioneers', also sell tea from estates owned by its own holding company along with lots of tea from clients.

Lot Data: For each lot of tea, the catalog lists the name of the estate, the lot number, the quantity, the appraised value (or "valuation," as the brokers called it), the warehouse address, and some other information. The final catalog is sent to all registered bidders (currently 37 in total).

Quality Ratings: Auctioneers taste the tea to be auctioned off themselves to judge the quality of each lot. Since these evaluations are only used by the auction houses and are never shared with buyers or sellers, they are not chosen in a strategic way. Most of the time, the tasters (auctioneers) write detailed comments about how the tea leaves and liquor look or give the lot an alpha-numeric score. With the help of the tea-tasters of the auctioneer, I used these notes to make an index of quality rating and gave each lot a number score between 1 and 10. I use this index to rank the quality of tea in my experiments. These quality ratings can be used to control for quality differences between lots that can't be seen. This is a very useful part of my data set.

Winning Bids and Number of Bidders: Aside from the catalog data and quality ratings, the auction house also provided hand-collected data on the winning bid for each lot, and the number of bidders for each auction. The auctioneer actively collected the data on the number of bidders to conduct market research about the future demand of tea due to local consumer demand and demand from exporters.

Summary Statistics: My main estimation sample comprises data from 2495 auctions. Summary statistics for these auctions are presented in Table 3. In total, 1340.64 tons of tea were auctioned off at an average price of 223.19 Tk per kilogram of tea. These 2495 auctions generated sale of approximatley 300 million taka in total.

Table 3: **Summary Statistics**

Average Appraised Value	212.55 Tk
Average Sale Price	223.19 Tk
Average Lot Size	537.33 Kg
Median Quality Rating	6.5
Number of Tea Estates	139
Average Number of Bidders	4.16
Total Volume	1340.64 Ton
Total Sales	299.22 Million Tk
N	2495

The average appraised value trails the average sales price by 10.64 Tk. In my data, I observe that there are 17 auctions where the sales price is lower than the valuation price, vindicating the fact that the valuation price is not a (binding) reserve price. The number of tea estates which sold their tea through the auctions that are detailed in my data is 139, compared to the total number of 167 tea estates in Bangladesh. The average number of bidders in an auction in my dataset is 4.16.

5 Reduced Form Analysis

I anticipate that the number of bidders, the valuation price (appraised value), the quality rating, the size of the lot affect the winning bid. In general, the rules of the auction, the type of good being sold, the characteristics of the bidders, and the characteristics of the industry as a whole all affect how the winning bid and the number of bidders are related. Theoretical auction models suggest that the number of bidders will cause the expected winning bids to go up. The simplest case is when firms are risk-neutral, the assumptions of the Independent Private Value model hold, and the distribution of bidder values follows a well-defined distribution function. Then, as the number of bidders goes up, the winning bid should go up by the expected second-order statistic from the value distribution (Brannman et al. (1987)).

The effect of a reference or benchmark price on the bidding behaviour has largely remained unexplored. Dholakia and Simonson (2005) and Wolk and Spann (2008) examined the effect of reference prices on consumer bidding behavior in online auctions for retailing. They found significant effect of the reference price on bidding behaviour. In their study the impact of the reference price depended on the credibility of the advertiser or the platform which provided the reference price. In general, in case of a trusted advertiser a higher reference price led to higher bids from the buyers.

In case of Chattogram tea auctions, a high number of bidders for a particular auction should indicate high demand for that particular lot. Thus, in line with the theoretical predictions, we should expect that the number of bidders in an auction affects the winning bid. In particular, I hypothesize that the higher the number of bidders for a tea lot, the higher is the winning bid. Also, I anticipate that the reference price, in this case the 'appraised valuation' made by the auctioneer, would also affect the bidding behaviour. The auctioneers receive commission from both the buyers and the tea-estates. If they set too high a price, the particular lot might not get sold and they receive no commission. On the other hand, if the lot is sold at too low a price, the broker would receive lower commission from the tea estates, as the commission is set at a percentage of the selling price. Hence, the auctioneers should have an incentive to come up with a credible figure for the reference price. The bidders, anticipating a credible reference price from the auctioneer, would thus put higher value for a lot assigned with higher valuation. Thus, I hypothesize that the higher the reference price (or the valuation), the higher would be winning bid.

To test my hypotheses I run the following regression.

$$\omega_i = \beta_0 + \beta_1 bidders_i + \beta_2 val_i + \beta_3 size_i + \beta_4 ratings_i + u_i \tag{1}$$

where ω_i denotes the winning bid in auction. I denote the number of bidders in auction i by $bidders$. The appraised value or the reference price is denoted by val_i. The quality ratings and size (in kg) for each lot are denoted by $ratings_i$ and $size_i$. In addition to the above regression I run another regression controlling for tea-estate specific fixed effects and auction-day specific fixed effects. The results of these two regressions are presented in Table 4.

We see that an additional bidder would raise the winning bid by 5.51 Tk which is 2.47% of the average winning bid in my data. Also, on average, if the appraised value (or the reference

Table 4: **Regression Results for Winning Bids**

	(OLS) Winning Bid	(FE) Winning Bid
Number of Bidders	5.551***	5.273***
	(0.174)	(0.161)
Valuation	1.034***	1.041***
	(0.00417)	(0.0113)
Size	-0.00353***	-0.00308***
	(0.000877)	(0.00114)
Ratings	-0.370***	-0.458***
	(0.0796)	(0.142)
Constant	-9.297***	-5.626
	(0.890)	(3.565)
Estate FEs	No	Yes
Auction Day FE	No	Yes
Observations	2,495	2,495
R-squared	0.987	0.990

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

price) goes up by 1 Tk the winning bid also goes up by 1 Tk. The coefficients of both these variables are also statistically significant. Thus, the regression results support my hypotheses. Also, the coefficient of the size variable is negative which is line with the fact that the higher the quantity the lower the price. Surprisingly, the coefficient of ratings is negative. This might be due to the fact that once I control for number of bidders, valuation, and size, the ratings do not add much to explaining the variation in the winning bids. However, the correlation coefficient between winning bids and ratings is 0.61, which is positive and statistically significant.

6 Model

In this section I develop a structural empirical model to study the oral, ascending tea auctions in Chattogram under the symmetric, independent private values (IPV) assumption.

Suppose there are $\mathcal{N}$ potential buyers for a given lot. The symmetric IPV assumption implies (a) the valuation v_i (or willingness to pay for the lot) of bidder i, is privately known to the bidder; (b) the bidders' valuations $v_1, \ldots, v_N$ are drawn independently and identically from some underlying distribution or distributions that are common knowledge. In this standard framework (see Krishna (2002)), each bidder i is assumed to have a valuation, or value v_i for a given lot of tea. The bidder's payoff from winning this lot at a price of P equals $v_i - P$.

It is well known that for ascending auctions in the IPV setting (see Klemperer (1999)), in Bayesian equilibrium a player's bid (or price at which he quits the auction) equals his valuation. The winning bid or sale price is therefore equal to the second highest valuation out of $(v_1, \ldots, v_N)$. If I assume that the bidders' strategies in my data set are not dynamic but lot-specific as in the above description, it then follows that the winning prices are realizations of the second (i.e., second-highest) order statistic.

Now, consider a sequence of oral ascending auctions $t = 1, \ldots T$ each having no reserve price, so all $\mathcal{N}_t$ potential bidders participate. Suppose only the winning bid W_t and $\mathcal{N}_t$ are recorded. Thus, I receive the sequence $\{(w_t, \mathcal{N}_t)\}_{t=1}^{T}$.

At oral, ascending-price auctions within the IPVP, assuming the clock model of Milgrom and Weber (1982), each non-winning bidder reveals his valuation truthfully. Now, under clock model assumptions, the cumulative distribution function $F_W(w)$ of the winning price W is the cumulative distribution function of the second-highest valuation $V_{(2:\mathcal{N})}$.

Now, as mentioned in Athey and Haile (2002), I can then use the following relation of the order statistics:

$$F_v^{(n-1:n)}(v) = \sum_{j=n-1}^{n} \begin{pmatrix} n \\ j \end{pmatrix} F_v(v)^j \left(1 - F_v(v)\right)^{n-j}$$

when solving for each $\hat{F}_v(v)$. Here, the order statistics are indexed from lowest to highest so that, $F_v^{(n:n)}(v)$ is the maximum (or the first-order statistics).

For the second-order statistics, the above equation simplifies as following:

$$F_W(w) = F_V(w)^{\mathcal{N}} + \mathcal{N} F_V(w)^{\mathcal{N}-1} \left(1 - F_V(w)\right)$$

This equation is intuitive: the first term in the right hand side is the probability that all the valuations are less than w (in which case the second highest one is also less than w). The second term is the probability that one of the valuations is greater than w and the rest are less than w, in which case the second highest valuation is again less than w. This second term includes the multiplicative factor $\mathcal{N}$, which counts the number of ways in which the second event can happen. Adding these two terms, gives total probability of the second highest valuation being less than w (i.e. the cumulative probability distribution of the second order statistics) in terms of the parent distribution, $F_v(V)$.

Thus, in the absence of a minimum bid price, when the number of potential bidders $\mathcal{N}$ is fixed and known, the empirical distribution of winning bids $\{w_t\}_{t=1}^{T}$ for a sample of T auctions, can be used to estimate the cumulative distribution of valuations by solving the above equation.

6.1 Likelihood Estimation

Now, suppose that the logarithm of the valuation for a potential bidder $logV$ is distributed normally with mean μ and variance σ^2.

Assuming that valuations are independent and identically distributed, I first derive the cumulative distribution function of winning bid W_t in terms of the cumulative distribution function $\Phi(z)$ for a standard normal random variable as well as $\mathcal{N}_t$.

To begin, let us first order the valuations of the bidders

$$V_{(1:\mathcal{N}_t)} > V_{(2:\mathcal{N}_t)} > \ldots > V_{(\mathcal{N}_t:\mathcal{N}_t)}.$$

where $V_{(i:\mathcal{N}_t)}$ denotes the i^{th} order statistic. Now, the winning bid, W_t is $V_{(2:\mathcal{N}_t)}$. As mentioned earlier, the cumulative distribution function of the second-highest order statistic is given by:

$$F_W\left(w \mid \mathcal{N}_t\right) = F_V(w)^{\mathcal{N}_t} + \mathcal{N}_t F_V(w)^{\mathcal{N}_t - 1}\left(1 - F_V(w)\right)$$

By rearranging the terms on the right hand side, this can be written as,

$$F_W\left(w \mid \mathcal{N}_t\right) = \mathcal{N}_t F_V(w)^{\mathcal{N}_t - 1} - \left(\mathcal{N}_t - 1\right) F_V(w)^{\mathcal{N}_t}$$

With the assumption of log-normal distribution for the valuations,

$$F_W(w \mid \mathcal{N}_t) = \mathcal{N}_t \Phi\left(\frac{\log w - \mu}{\sigma}\right)^{\mathcal{N}_t - 1} - (\mathcal{N}_t - 1)\,\Phi\left(\frac{\log w - \mu}{\sigma}\right)^{\mathcal{N}_t}$$

To derive the probability density function of W_t (probability density of the second-highest order statistic), I differentiate the cumulative distribution function with respect to w. So,

$$\begin{aligned} f_W(w \mid \mathcal{N}_t) = & \frac{\mathcal{N}_t(\mathcal{N}_t - 1)}{w\sigma} \Phi\left(\frac{\log w - \mu}{\sigma}\right)^{\mathcal{N}_t - 2} \\ & \left[1 - \Phi\left(\frac{\log w - \mu}{\sigma}\right)\right] \phi\left(\frac{\log w - \mu}{\sigma}\right) \end{aligned}$$

Using data on the the winning bid $\{w_t\}_{t=1}^T$ and the number of bidders $\{\mathcal{N}_t\}_{t=1}^T$ I can now write the likelihood function. The probability density function relevant to the derivation of the likelihood function is that of the second-highest order statistic. The likelihood function is:

$$L = \prod_{t=1}^{T} \frac{\mathcal{N}_t(\mathcal{N}_t - 1)}{w_t \sigma} \Phi\left(\frac{\log w_t - \mu}{\sigma}\right)^{\mathcal{N}_t - 2} \left[1 - \Phi\left(\frac{\log w_t - \mu}{\sigma}\right)\right] \phi\left(\frac{\log w_t - \mu}{\sigma}\right).$$

Thus, the logarithm of the likelihood function is

$$\begin{aligned} \mathcal{L}(\mu, \sigma; \boldsymbol{w}, \mathcal{N}) = & \sum_{t=1}^{T} \log\left[\mathcal{N}_t(\mathcal{N}_t - 1)\right] + \\ & \sum_{t=1}^{T} (\mathcal{N}_t - 2) \log\left[\Phi\left(\frac{\log w_t - \mu}{\sigma}\right)\right] + \\ & \sum_{t=1}^{T} \log\left[1 - \Phi\left(\frac{\log w_t - \mu}{\sigma}\right)\right] + \\ & \sum_{t=1}^{T} \log\left[\phi\left(\frac{\log w_t - \mu}{\sigma}\right)\right] - \\ & \sum_{t=1}^{T} \log w_t - T \log(\sigma) \end{aligned}$$

In addition to using the winning bids and the number of bidders in each auction t, I also use the log of the 'appraised value' (reference price) made by the auctioneer as a covariate x_t. Using these data, I estimate the parameters μ and σ by the method of maximum likelihood where $\mu(x_t)$ equals $\gamma_0 + \gamma_1 x_t$. The estimate of $\left(\hat{\gamma}_0^{\text{ML}}, \hat{\gamma}_1^{\text{ML}}, \hat{\sigma}^{\text{ML}}\right)^\top$ is provided in table 5.[3]

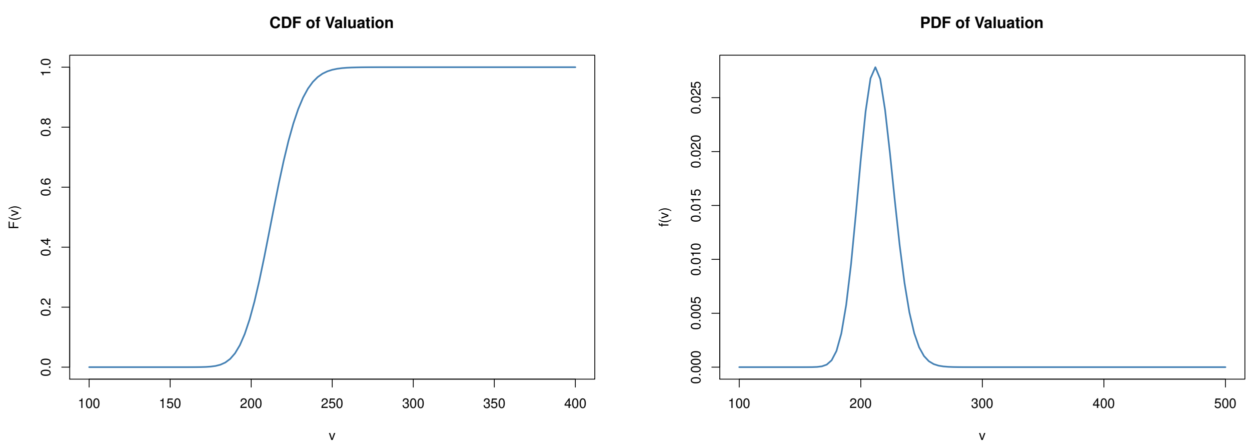
[3] The standard errors were calculated by the taking the square root of the diagonal elements of the inverse of the Hessian matrix.

Table 5: **ML Estimate of the Parameters of the Log-Normal Distribution**

Parameter	ML Estimate
$\hat{\gamma}_0$	0.1763 (0.0194)
$\hat{\gamma}_1$	0.9726 (0.0036)
$\hat{\sigma}$	0.0675 (0.0010)

The following figure shows the cumulative as well as probability densities of valuation with maximum likelihood estimates of the parameters of the log-normal distribution at the mean appraised value observed in the data.

Figure 3: Distribution of Valuation with Maximum Likelihood Estimates of the Parameters



I am going to compare the likelihood estimate of the distribution of valuations with distributions estimated by least-squares, and non-parametric methods. This comparison would help us to decide how robust the results are from the likelihood estimation.

6.2 Least Square Estimation

Order statistics are preserved under location-scale transformation. The normal random variable $logV$ lives within the location-scale family of random variables. Thus, the i^{th} order statistics of $logV$ can be written as follows:

$$\log V_{(i:\mathcal{N}_t)} = \mu + \sigma Z_{(i:\mathcal{N}_t)}$$

where $Z_{(i:\mathcal{N}_t)}$ is the i^{th} largest order statistic from a sample of $\mathcal{N}_t$ independent and identically distributed standard normal random variables and where the following holds: $Z_{(1:\mathcal{N}_t)} \geq Z_{(2:\mathcal{N}_t)} \geq \ldots Z_{(\mathcal{N}_t:\mathcal{N}_t)}$. I know then, for the second-highest order statistic, the following must be true:

$$\begin{aligned}
&\mathbb{E}[\log(W_t) \mid \mathcal{N}_t] \\
&\quad = \mu + \sigma \mathbb{E}\left[Z_{(2:\mathcal{N}_t)}\right] \\
&\quad = \mu + \sigma \int_{-\infty}^{\infty} z\mathcal{N}_t(\mathcal{N}_t - 1)\, \Phi(z)^{\mathcal{N}_t - 2}[1 - \Phi(z)]\phi(z)dz
\end{aligned}$$

I compute the integral above numerically using trapezoidal quadrature rule for each $\mathcal{N}_t$ in my data.

The regression is then constructed by exploiting the location-scale property, mentioned above. In particular, the regression equation is :

$$\begin{aligned}
\log(W_t) &= \mathbb{E}\left[\log(W_t) \mid Z_{(2:\mathcal{N}_t)}\right] + \epsilon_t \\
&= \mu + \sigma \mathbb{E}\left[Z_{(2:\mathcal{N}_t)}\right] + \epsilon_t
\end{aligned}$$

where $\mathbb{E}(\epsilon_t \mid \mathcal{N}_t)$ is zero, but the variance of ϵ_t depends on $\mathcal{N}_t$. I can now estimate the parameters μ and σ by the method of least squares where $\mu(x_t)$ equals $\gamma_0 + \gamma_1 x_t$., where x_t denotes log of the valuation provided by the auctioneer.

The least squares estimate of $\left(\hat{\gamma}_0^{\text{LS}}, \hat{\gamma}_1^{\text{LS}}, \hat{\sigma}^{\text{LS}}\right)^\top$ is provided in table 6.The least-squares estimates of the parameters of the log-normal distribution is almost the same as those obtained from the likelihood estimation.

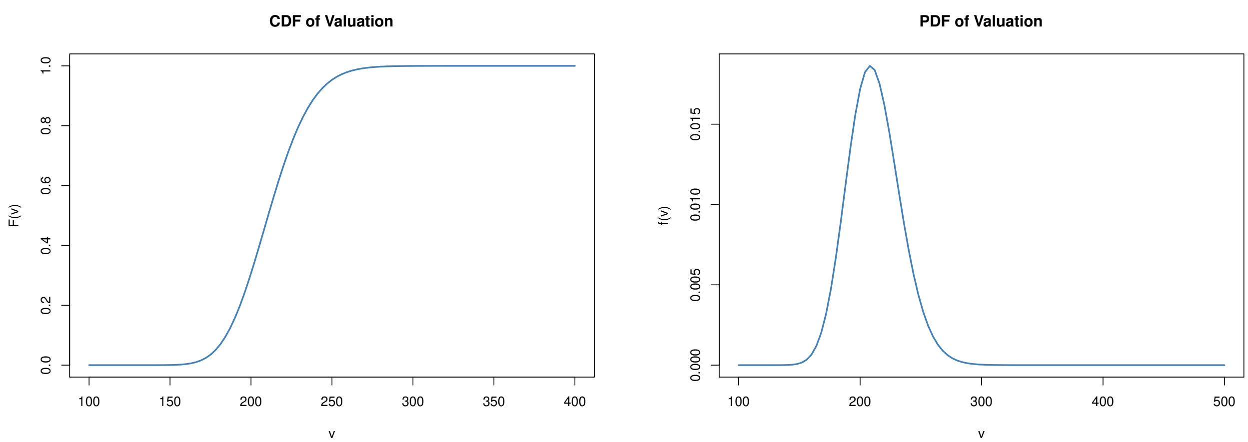
Figure 4 shows the cumulative as well as probability densities of Valuation with least squares estimates of the parameters of the log-normal distribution at the mean appraised value observed in the data.

As evidenced from Table 5 and Table 6, the least-squares estimates of the parameters of

Table 6: **LS Estimate of the Parameters of the Log-Normal Distribution**

Parameter	LS Estimate
$\hat{\gamma}_0$	0.1415 (0.0211)
$\hat{\gamma}_1$	0.9771 (0.0040)
$\hat{\sigma}$	0.1022 (0.0033)

Figure 4: Distribution of Valuation with Least Squares Estimates of the Parameters



the log-normal distribution are very close to those of likelihood estimates. One can use the results from either of these two parametric estimation methods.

7 Non-parametric Estimation

In this section, I estimate the distribution of the valuation of the bidders non-parametrically. Under the IPVP assumptions, the dominant-strategy, equilibrium-bid function at Vickrey auctions is

$$B_i = \beta(V_i) = V_i \quad i = 1, \ldots, \mathcal{N}.$$

Thus, the probability density function of observed bids is related to that of valuations by

$$f_B^0(b) = f_V^0(v),$$

so

$$F_V^0(v) = F_B^0(v)$$

where the superscripts "0" denote the true population values. The distribution of bids identifies the distribution of valuations.

Suppose we have a sample of T independent auctions of an identical object with identical number of bidders $\mathcal{N}$. Index the auctions by $t = 1, \ldots, T$. A natural way to estimate $F_V^0(v)$ would be to substitute the sample analogue for the population quantity, so one estimator of $F_V^0(v)$ is defined by

$$\hat{F}_V(v) = \frac{1}{T\mathcal{N}} \sum_{t=1}^{T} \sum_{i=1}^{\mathcal{N}} \mathbf{1}\left(B_{it} \leq v\right).$$

Even if only the winning bids were observed at a Vickrey auction, $F_V^0(v)$ would be non-parametrically identified. Since the winning bid is equal to the second order statistics,

$$W = V_{2:\mathcal{N}}$$

we can write,

$$F_W^0(w) = \varphi\left[F_V^0(w) \mid \mathcal{N}\right]$$

where $\varphi(\cdot \mid \mathcal{N})$ is a known, strictly monotonic function. Thus, $F_V^0(v)$ is identified from the distribution of observed winning bids $F_W^0(w)$, when $\mathcal{N}$ is known.

As mentioned before, we can use the following relation of the order statistics to estimate $F_V(w)$:

$$F_W(w) = F_V(w)^{\mathcal{N}} + \mathcal{N} F_V(w)^{\mathcal{N}-1}\left(1 - F_V(w)\right)$$

Monotonicity of the relation between $F_W(w)$ and $F_V(w)$ makes numerical solution particularly simple. Now, under standard regularity conditions, which are satisfied in this theoretical model of an auction, the asymptotic analysis of $\hat{F}_W(v)$ is well understood. In particular,

$$\sqrt{T}\left[\hat{F}_W(v) - F_W^0(v)\right] \xrightarrow{\text{d}} \mathcal{N}\left\{0, \mathcal{V}_W\left[F_V^0(v) : \mathcal{N}\right]\right\}$$

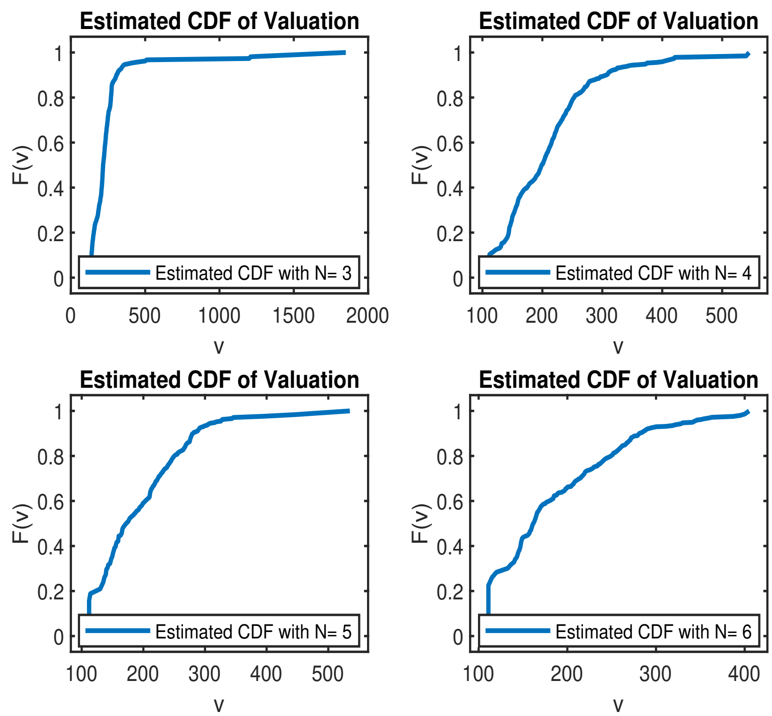
where

$$\begin{aligned} \mathcal{V}_W\left[F_V^0(v); \mathcal{N}\right] &= F_W^0(v)\left[1 - F_W^0(v)\right] \\ &= \varphi\left[F_V^0(v) \mid \mathcal{N}\right]\left\{1 - \varphi\left[F_V^0(v) \mid \mathcal{N}\right]\right\} \end{aligned}$$

Note that when there is exogenous variation in the number of bidders, there will be as many different estimators of $F_V(v)$ available as there are different values of $\mathcal{N}$ in the data.

My dataset includes the winning bid, w_t, as well as the number of bidders $\mathcal{N}$ participating at an auction, t. Hence, I could estimate the distribution of the valuation following the procedure mentioned above. Figure 5 shows the estimated cumulative distribution functions of the valuation for different number of bidders observed in the data.

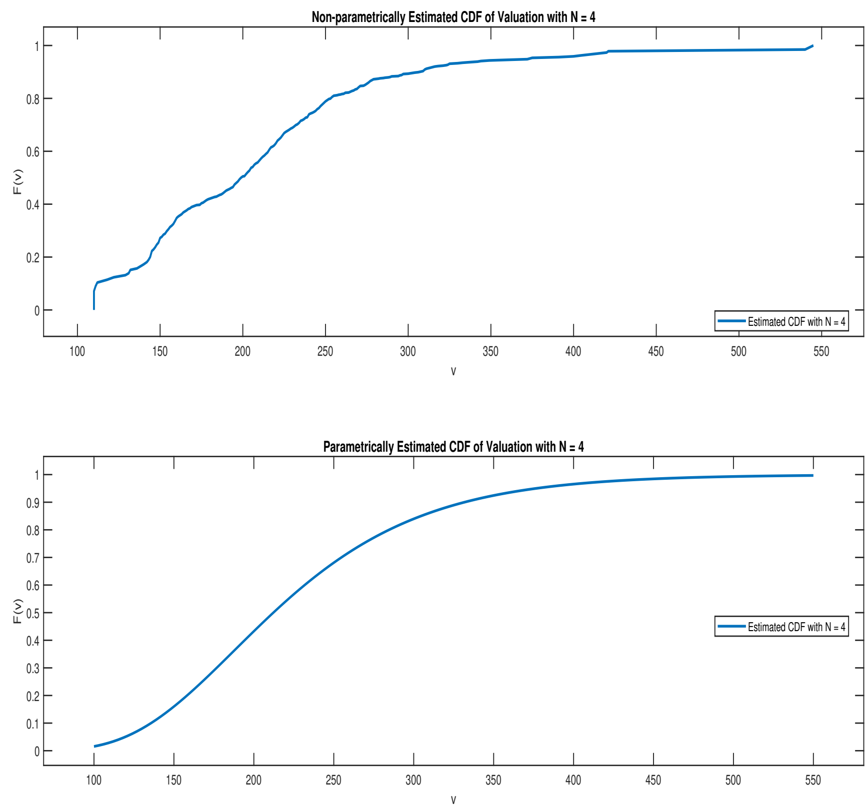
Figure 5: Non-parametric Estimation of Distribution of Valuations



Finally, I compare the estimated distribution of valuations from the parametric (likelihood estimates) and non-parametric methods. As mentioned before, there will be as many different non-parametric estimators of $F_V(v)$ available as there are different values of $\mathcal{N}$ in the data. For comparison with parametric estimates, I estimate the cdf of valuations using likelihood methods for the subset of data where the number of bidders is 4 (which is close to the average number of bidders in an auction in the dataset, which is 4.16). I then compare this distribution with non-parametrically estimated cdf of valuation with $N = 4$.

Figure 6 shows the estimated distribution of valuations using both the parametric and non-parametric methods. The parametrically estimated cdf of valuations corresponds to a log-normal distribution at the mean appraised value observed in the subset of data where $N = 4$. The two cdfs in Figure 6 look remarkably similar.[4] This gives further support to the robustness of our likelihood estimates of the distribution of valuations.

Figure 6: Comparing Parametric and Non-parametric Estimates of Distribution of Valuations



Since the parametric and non-parametric estimates of the distribution of valuation are similar, the results of the counterfactual exercises based on the distribution of valuations would almost be the same, no matter which estimates are used for calculation. I am going to use the likelihood estimates of the distribution of valuations for my counterfactual exercises.

[4]Comparing likelihood and non-parametric estimates for cdf of valuations using different subsets of data with varying N gives similar results

8 Defence of Symmetric IPVP Assumption

Anecdotal evidence suggests that English auctions are typically used in environments where little information useful to all of the bidders concerning the value of the object is revealed in the course of the auction. Sales of oil and gas leases, for example, are not undertaken using English auctions because the proprietary information of any particular bidder concerning the probability of discovering oil (and thus the value of the lease) would be revealed in the course of bidding. That English auctions are used to sell tea lends support to the IPVP assumption.

Moreover, during the period considered in this analysis (November 2021 to July 2022) considerable price stability existed in the tea auctions. So, any asymmetries in expectations concerning future prices were unlikely important.

Explanation for differences in bidding behaviour might arise due to differences in further marketing costs for the participating bidders (who are mostly wholesalers), which are likely individual-specific effects. However, I assume that, for a given sale, these differences are random and independent across potential bidders as well as being identically distributed.

9 Bidder Surplus

In this section, I calculate the average surplus that a bidder receives by winning a particular lot using the parametric estimates of the distribution of valuation. For the purpose of this exercise, I use the estimates for the log-normal distribution of the valuation.

A bidder's surplus at an auction t is

$$CS_t = v_t^{(1:n_t)} - p_t$$

where p_t denotes the price that a buyer actually paid and $v_t^{(1:n_t)}$ denotes the first-order statistic. Since $v_t^{(1:n_t)}$ is not observed, following Song (2004) I estimate an expected consumer's surplus as follows:

$$E\left[CS_t \mid V_t^{(2:N_t)} = v_t^{(2:n_t)}\right] = \int_{v_t^{(2:n_t)}}^{\infty} \frac{f(x)}{1 - F\left(v_t^{(2:n_t)}\right)} \cdot x dx - p_t.$$

where $v_t^{(2:N_t)}$ is the second-order statistic, which is given by the observed winning bid for each auction. Hence, the first term in the right hand side is the expected valuation, given the valuation is higher than the winning bid. I subtract the winning bid from this to calculate the surplus.

The estimate of the average bidder's surplus is Tk 16.15 per kilogram with standard error of Tk 4.67. This means that for a lot weighing 500 kg, the estimated surplus is Tk 8075.01. The average surplus in an auction is 7.24% of the average winning bid.

This estimate of bidder surplus may indicate how much bidders can benefit from the auction, because tea is seldom transacted outside of the auction center.

10 Revenue Equivalence

Within the symmetric IPV model, under the assumption that potential buyers are risk neutral with respect to winning the object for sale, a remarkable result has been obtained in auction theory: revenue equivalence (in expectation). That is, if the same object were sold under two different institutions, then the average winning bid at the English auction would equal the average winning bid at the Dutch auction. This result, was first outlined by Vickrey (1961) and then proved in general by Myerson (1981). In this section, we want to test if the revenue equivalence theorem holds in case of the Chattogram tea auctions.

Under the symmetric IPVP, Dutch auctions and first-price, sealed-bid auctions are strategically equivalent. How should a risk-neutral potential bidder behave at either a Dutch or a first-price, sealed-bid auction? As shown in Milgrom (2004), in a symmetric Bayes-Nash equilibrium of a first-price, sealed bid auction with $\mathcal{N}$ symmetric bidders, where valuations are i.i.d. random variables, each bidder i will bid

$$\sigma(v_i) = v_i - \frac{\int_0^v F_V(u)^{(\mathcal{N}-1)} du}{F_V(v)^{(\mathcal{N}-1)}}$$

where $\sigma(v_i)$ denotes the bidding function of player i. Note that $\sigma(v_i)$ depends on the bidder's own valuation, v_i and $\mathcal{N}$ as well as on the distribution of valuation, $F_V(\cdot)$. The bidder submits a bid that is less than his valuation, a phenomenon know as 'bid-shading'. The winner at Dutch and first-price, sealed-bid auctions will be the bidder with the highest valuation $v_{(1:\mathcal{N})}$.

10.1 Counterfactual Estimates of First-Price Auction Bids

In my data, I observe the winning bids as well as the number of bidders. I use those information with my parametric estimates of the log-normal distribution to simulate auctions under first-price rule. I proceed as follows. First, for each auction in my data I generate random numbers from the log-normal distribution of the valuation with my parameter estimates, the appraised value of the lot, and the observed number of bidders in that auction. Then for each valuation, I compute corresponding bid numerically. After that, I take the highest bid as the winning bid. I replicate this procedure 1000 times and each time I get a winning bid. I then take the average of these 1000 winning bids to get a measure for the winning bid under a first-price auction for the corresponding lot.

For example, for one of the lots in my dataset I observe that the winning bid is Tk 210 with 4 bidders participating in the English auction. To get a counterfactual winning bid under a first-price auction in this case, I first generate a 4 x 1000 matrix of random numbers (4 bidders and 1000 simulated auctions) from a log-normal distribution with my estimated parameters and the appraised value of the lot, which in this case is Tk 190. Once I have this random matrix of valuations, I can then compute the corresponding bid (numerically) under first-price auction using the formula stated above. Then, for each of the 1000 simulated auctions I take the highest bid as the winning bid. After that I take the average of those 1000 winning bids to get the corresponding counterfactual winning bid under first-price auction. The following table 7 shows bids from two simulated first price auctions with different number of bidders.

As we can see, the bids under the first-price auction are lower than the valuations. The bidders are shading their bids. Also, the optimal bids are dependent on the auctions. As predicted, the lower the number of bidders the higher the magnitude of shading.

I then compute the average winning bids across all the simulated 2495 (the total number of lots observed in the data) first price auctions. The average winning bid under the simulated first-price auctions is Tk 220.71 (standard deviation of Tk 61.38), which is less than the observed average winning bid under the English auction which is Tk 222.89 (standard deviation of Tk 65.22). A paired t-test also confirms that the difference is statistically significant.

Thus, it appears that the English auction, at which an efficient allocation always obtains, also garnered more revenue than the first-price (Dutch) auction. The administrators of the tea auctions appear to have made a good choice of selling mechanism.

Table 7: **Simulated Bids in First-Price Auction**

Number of Bidders	Appraised Value of the Lot	Bidder Valuation (v_i)	Bids under first-price auction (b_i)
n = 3	240	268.88	246.55
		285.67	252.11
		235.91	226.47
n = 4	190	197.59	190.05
		185.18	180.36
		199.16	191.18
		208.36	197.21

11 A Model with a Reserve Price

Consider $\mathcal{N}$ potential bidders who are vying to purchase an object at a Vickrey auction within the IPVP. At Vickrey auctions, the dominant bidding strategy is to bid one's valuation. So bidder i 's bid B_i is related to his valuation V_i according to

$$B_i = \beta(V_i) = V_i \quad r \leq V_i$$

where r is the reserve price, the minimum price which must be bid. Clearly, those potential bidders for whom V_i is less than r will choose not to participate at the auction; they will not appear in data collected.

Suppose that a collection of homogeneous objects is sold individually at a sequence of T different Vickrey auctions where the same reserve price r has been imposed. Consider that at auction t, only the winning bid w_t as well as a measure of the number of potential competitors $\mathcal{N}_t$ is recorded.

Since there exists a potentially binding reserve price, we must incorporate this fact into the likelihood function. With a reserve price, only bids over the reserve price are observed. Hence, in the presence of a binding reserve price, we can only estimate a truncated distribution: the distribution of valuations which are above the reserve price. I need to divide the probability density function of the second-highest order statistic (the winning bid) by the survivor function evaluated at the reserve price r. Thus, the distribution of the winning bids (second-order statistic) is:

$$f_{W_t}(w) = \frac{\mathcal{N}_t\left(\mathcal{N}_t - 1\right) F_V(w)^{\mathcal{N}_t - 2}\left[1 - F_V(w)\right] f_V(w)}{\left[1 - F_V(r)^{\mathcal{N}_t}\right]}$$

which implies that the likelihood function is

$$\prod_{t=1}^{T} \frac{\mathcal{N}_t\left(\mathcal{N}_t - 1\right) F_V(w)^{\mathcal{N}_t - 2}\left[1 - F_V(w)\right] f_V(w)}{\left[1 - F_V(r)^{\mathcal{N}_t}\right]},$$

and that the logarithm of the likelihood function is

$$\sum_{t=1}^{T} \log\left[\mathcal{N}_t\left(\mathcal{N}_t - 1\right)\right] + \sum_{t=1}^{T}\left(\mathcal{N}_t - 2\right)\log\left[F_V\left(w_t\right)\right] + \sum_{t=1}^{T}\log\left[1 - F_V\left(w_t\right)\right] + \sum_{t=1}^{T}\log\left[f_V\left(w_t\right)\right] - \sum_{t=1}^{T}\log\left[1 - F_V(r)^{\mathcal{N}_t}\right]$$

11.1 Optimal Reserve Price

Riley and Samuelson (1981) have shown that within the IPVP the selling mechanism that maximizes the seller's expected gain is a selling mechanism where a reserve price ρ is chosen optimally. What is the optimal reserve price ρ^* ?

The utility to the seller is the sum of the expected utility of retaining the object for sale and the expected revenue when the object is sold at the auction. Hence, the total utility for the seller is

$$v_0 F_V(r)^{\mathcal{N}} + \mathcal{N}\int_r^{\bar{v}} \left[u f_V(u) + F_V(u) - 1\right] F_V(u)^{\mathcal{N}-1} du.$$

where v_0 is the seller's valuation of the object at auction.

Differentiating this with respect to the reserve price r gives the following first-order condition that should hold when r equals ρ^* :

$$\mathcal{N} v_0 F_V(r)^{\mathcal{N}-1} f_V(r) - \mathcal{N}\left[r f_V(r) + F_V(r) - 1\right] F_V(r)^{\mathcal{N}-1} = 0.$$

Factoring out the common terms $\mathcal{N}, F_V(r)^{\mathcal{N}-1}$, this becomes

$$v_0 f_V(r) - r f_V(r) - F_V(r) + 1 = 0.$$

Therefore, within the symmetric IPVP, the optimal selling mechanism requires that the optimally-chosen reserve price ρ^* solves the following equation:

$$\rho^* = v_0 + \frac{\left[1 - F_V\left(\rho^*\right)\right]}{f_V\left(\rho^*\right)}$$

Clearly, to calculate ρ^* we require information concerning $F_V(v)$, the distribution of valuations.

For the purpose of the calculation of the optimal reserve price I assume, that the distribution of valuation is from the Weibull family with the following probability density function

$$f_V(v; \boldsymbol{\theta}) = \theta_1 \theta_2 v^{\theta_2 - 1} \exp\left(-\theta_1 v^{\theta_2}\right) \quad v \geq 0, \theta_1 > 0, \theta_2 > 0$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2)^\top$.

I use the likelihood function mentioned above to get the maximum-likelihood estimate of the parameters of the Weibull distribution.

I now characterize the asymptotic distribution of the optimal reserve price. I begin by defining

$$g\left(\hat{\rho}, \theta_1, \theta_2\right) = \hat{\rho} - v_0 - \frac{1}{\hat{h}_V\left(\hat{\theta}_1, \hat{\theta}_2\right)} = 0$$

where $\hat{h}_V(\cdot)$ denotes the hazard function of the Weibull distribution. For simplicity, let $\boldsymbol{\theta}^{\mathbf{0}}$ denote $\left(\theta_1^0, \theta_2^0\right)$. Now, using a first-order, Taylor-series expansion, I can write

$$\left(\hat{\rho} - \rho^0\right) = \frac{-\nabla_{\boldsymbol{\theta}} g\left(\rho^0, \boldsymbol{\theta}^0\right)}{g_\rho\left(\rho^0, \boldsymbol{\theta}^0\right)} \left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0\right)$$

where $g_\rho(\cdot)$ denotes the derivative of $g(\cdot)$ with respect to ρ. Thus,

$$\sqrt{T}\left(\hat{\rho} - \rho^0\right) \xrightarrow{\mathrm{d}} \mathcal{N}\left[0, \mathbf{a}^\top \mathcal{I}^{-1}\left(\boldsymbol{\theta}^0\right)\mathbf{a}\right]$$

where

$$\mathbf{a} = \frac{-\nabla_{\boldsymbol{\theta}} g\left(\rho^0, \theta^0\right)}{g_\rho\left(\rho^0, \boldsymbol{\theta}^0\right)}$$

and where $\mathcal{I}^{-1}$ denotes the inverse of Fisher's information matrix.

There is no binding reserve price in Chattogram tea auctions. In my data, the lowest of the appraised value set by the auctioneer is 95 Tk with an average of 212.94 Tk. The lowest winning bid is 110 Tk. I estimate my model presented above with a reserve price set at different levels: I set $r = 0$, $r = 95$, and $r = 110$. For each of these reserve price I get my model parameter estimates and the Hessian matrix. Using those, I calculate the optimal reserve price at different levels of minimum valuation by the bidders, v_0, given an initially set reserve price by the auctioneer. The results are presented in table 8.

Table 8: **Estimates of Optimal Reserve Prices**

r	v_0	$\hat{\rho}^*$
$r = 0$	$v_0 = 0$	70.81
	$v_0 = 95$	179.47
	$v_0 = 212.94$	281.72
$r = 95$	$v_0 = 95$	180.04
	$v_0 = 110$	193.08
	$v_0 = 212.94$	283.61
$r = 110$	$v_0 = 95$	179.94
	$v_0 = 110$	192.99
	$v_0 = 212.94$	283.64

The results suggest, if the reserve price is initially set at the minimum of the appraised value observed in the data ($r = 95$), and if the minimum valuation of the bidder is equal to 110 (which is the minimum winning bid observed in the data), the optimal reserve price is 193.08. On the other hand, if initially there is no reserve price ($r = 0$), as is the case in these tea auctions , and if the minimum valuation of the bidder is equal to 95 (which is the minimum appraised value observed in the data), the optimal reserve price is 179.47.

As we can see in Table 8, the optimal reserve price is higher the higher the minimum valuation, as predicted by the model.

12 Counterfactual: Multi-Unit Auctions

In this section, I extend my empirical results derived from the case of single-unit actions to the case of multiple units. Throughout my analysis I have assumed that the auctions are for single-unit of lots. However, in Chattogram tea auctions, a bidder generally wins multiple lots. A bidder who, in general, is a wholesaler cares about the the total number of lots won on a particular day. In fact, we can think of Chattogram tea auctions as a sequential auction, at which one unit (or lot containing several kilograms of the same tea) is sold at a time.

It is important to consider the difference between multi-object auctions and multi-unit auctions. At multi-unit auctions, the objects are assumed identical, so it matters not which unit a bidder wins but rather the aggregate number of units he wins, while at multi-object auctions the objects are assumed different, so a bidder is concerned about which specific object(s) he wins. It is evident that Chattogram tea auctions are a multi-unit auctions: (almost) identical tea lots are sold in sequential auction

These following characteristics describe multi-unit tea auctions in Chattogram:

(1) The auctioneer offers multiple lots of tea to a number of bidders.

(2) Bidders have a vector of unit valuations.

(3) Offers are made by bidders for the units they want.

(4) The units are distributed among bidders in accordance with an allocation rule: through oral ascending price auction.

The Generalized Vickrey Auction (GVA) is the multi-unit analogue of the second-price (Vickrey) auction. This mechanism is referred to as a Vickrey-Groves-Clarke mechanism due to Clarke (1971) and Groves (1973). Within the IPVP framework, (risk-neutral) bidders will always bid truthfully resulting in an efficient allocation.

The M units up for bid at a GVA are distributed to the bidders with the highest M reported valuations, or bids, but the amount that each winner must pay for the unit he wins is not determined by his reported bids (i.e., his bid for that unit), but rather by the bids of others. I first give a solved example of the algorithm before giving a broad description of it.

Consider the case where four units are for sale, and three bidders are participating at the auction; i.e., M is four and n is three. Suppose that the reported valuations by bidder 1 for the four units is V^1 equal $\{5, 4, 2, 0\}$, while for bidder 2 it is V^2 equal $\{6, 3, 1, 0\}$ and for bidder 3 it is V^3 equal $\{4.5, 2.5, 0, 0\}$. Given these reports, the aggregate-demand vector is $\{6, 5, 4.5, 4, 3, 2.5, 2, 1\}$. Thus, the four units will be allocated to bidders having reported valuations $\{6, 5, 4.5, 4\}$. The list and order of winners is $\{2, 1, 3, 1\}$. The price of the first unit sold will be V_4^{-2} or 2.5, which is the fourth-highest reported valuation, given that the reported valuations of bidder 2 have been eliminated from the aggregate-demand vector, while the price of the second unit sold will be V_4^{-1} or 2.5, and the price of the third unit will be V_4^{-3} or 3 , with the price of the last unit being V_3^{-1} or 3 . This last price warrants some explanation. In this case, bidder l wins the unit, but since he has already won a previous unit, one must use the third-highest of the remaining reported valuations in the aggregate-demand vector to price this unit.

In general, the algorithm for finding the prices paid at a GVA is the following: Let V^i denote the reported ordered valuations of bidder i and denote by V the ordered valuations of all bidders. Denote by V^{-i} the ordered valuations of all bidders, excluding bidder i. At a GVA, the M units are allocated to the bidders with the M highest reported valuations. If he wins ℓ_i units, then the price paid by bidder i for the first unit is the M^{th} highest reported valuation in V^{-i}, while the price paid by bidder i for the second unit is the $(M-1)^{\text{st}}$ highest reported valuation in $V^{-\imath}$, and so forth.

I now present a counterfactual exercise where I assume that the bidders in the Chattogram tea auctions participate in a Generalized Vickrey Auction. I consider an auction at which m identical units are to be sold sequentially. Below, I refer to the sale of a specific unit as a stage of the auction. I assume that all $n(\geq 2)$ potential bidders have weakly positive marginal utility for all units of the good for sale so that, in the absence of a reserve price, each potential bidder demands each of the m units. Again, a potential bidder draws his m independent valuations from the cumulative distribution function $F(v)$.

I use my estimates from the parametric estimation to get the (common) log-normal distribution of valuation for the bidders for each auction. I get the simulated valuations for each bidder for each of the lot for sale on any given auction day from the estimated distribution. I sort the valuations in a descending order for each bidder to ensure a weakly positive marginal utility for all units of the good for sale. This gives us the matrix of valuations V for n bidders and m units. Then, I write a program to compute the order of winners and the prices that they pay at a multi-unit Vickrey auction. In my dataset I observe 26 auction days. I simulate each auction day 1000 times to get the average winning bid for a lot and compare that average with that of the observed average winning bid for all the auctions held on a given auction day. The results from my counterfactual exercise are provided in table.

As we can see from Table 9, the average winning bid for each day of auction is higher in case of the simulated GVA. The average observed winning bid over the 26 auction day is Tk 226.81. On the other hand, the average winning bid in simulated GVA over the 26 auction days is Tk. 261.50. This difference in mean of Tk 34.69 is also statistically significant. [5] Hence, the results of the counterfactual simulations suggest that a multi-unit Generalized Vickrey Auction would bring in higher revenue for the auctioneer.

[5] A paired sample t-test for equality of means resulted a p-value close to zero

Table 9: **Comparison of English and Simulated VCG Auctions**

Auction Day	Number of Lots	Number of Bidders	Average Winning Bid	Average Winning Bid in Simulated GVA
1	56	9	258.1786	278.7074
2	39	8	274.0256	317.6254
3	77	12	249.6883	278.2549
4	97	16	239.6186	261.7444
5	102	13	241.7941	267.3539
6	88	17	244.3523	275.6695
7	79	11	229.5823	286.1991
8	98	21	238.449	257.3826
9	107	18	236.8972	273.4981
10	114	26	261.8421	287.8006
11	98	23	232.5408	282.9325
12	105	27	243.0381	268.4889
13	111	29	220.045	271.3999
14	106	24	239.4623	281.5584
15	121	29	236.8182	271.4257
16	107	25	235.1852	263.2037
17	108	27	228.5185	268.6457
18	103	23	228.8738	272.8943
19	96	21	213.7083	239.9352
20	103	29	219.3689	260.4473
21	103	23	203.7087	239.073
22	107	28	188.3832	231.4179
23	101	22	195.1485	230.0394
24	189	31	163.6931	200.9387
25	104	27	162.4712	185.6772
26	102	21	213.7745	246.6173

13 Conclusion

Given that market institutions are progressively replacing more conventional modes of exchange, auction theory has a wider range of applications in emerging country marketplaces. The practice of selling agricultural products at public auctions is gaining popularity since it offers openness and market information. A better understanding of these important market mechanism through empirical research would certainly add value to regulatory authorities of these markets.

This is perhaps one of the first studies of tea auctions in Bangladesh using the theory of auctions. I have tested structural models of behavior at these auctions. My study sheds light on the efficiency of the current mechanism where an oral, ascending price auction mechanism is in place.

I also want to mention the caveat that I have had to make some simplifying assumptions in order to make the model tractable, and therefore the results should be viewed as an approximation and should be interpreted with some caution. Nevertheless, one important policy implication arising from these results is the importance of accounting for multiple-units, and optimal reserve price in these tea auctions, which are shown to have a big influence on the revenue of the auctioneer.

Future research could extend the model proposed in this paper to study auctions used in other industries and settings. In case of Bangladesh tea auctions, a dynamic auction model may be estimated with a better dataset, where information on losing bids in each auction might be required.

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